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Mixed convection flow for unsteady oscillatory MHD second grade fluid in a porous channel with heat generation

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Abstract. In this paper, the mixed convection unsteady oscillatory MHD flow of a second grade fluid through a porous channel with heat generation is studied. It is assumed that the walls of the channel are porous so that injection/suction may take place. Using the flow assumptions the basic equations are reduced to ordinary differential equations which are solved analytically by perturbation technique. Flow and heat transfer results for a range of values of the pertinent parameters have been reported.

Keywords: Heat transfer; Second grade fluid; oscillating wall; analytical solutions; MHD; Porous medium.
Mathematics Subject Classification: 76xx, 76A05, 76A10, 83C30

1. INTRODUCTION

Heat transfer analysis in natural and mixed convection in vertical channels occurs in many industrial applications. It becomes a subject of many researches both numerically and analytically. Most of the interest in this subject is due to its applications in the design of cooling systems for electronic devices and in the field of solar energy collection. Some related articles on this area are [1],[2],[3],[4],[5],[6], and [7].

In the above quoted referee of natural and mixed convection flow in vertical channels are based on the hypothesis that the fluids are Newtonian. Besides of the fundamental and technological importance, theoretical studies of natural and mixed convection flows of non-Newtonian fluids in channels and tubes are of great important in most of the technological applications. Related papers on flow and heat transfer of non-Newtonian fluids in channels and tubes are [8],[9],[10],[11],[12],[13] and [7].

In the present paper we consider heat transfer of an electrically conducting second grade fluid in porous channels maintained at different temperatures in the presence of heat sources. Effects of pertinent parameters, such as the second grade parameter, the suction/injection parameter, the Grashof number, the Reynolds number, the Peclet number, the radiation parameter, Hartman number and the porous medium shape factor on velocity and temperature profiles are shown graphically and discussed.

The organisation of the paper is as follows. In section 2, the governing equations for the MHD flow with heat transfer of a second graded grade fluid are shown. In section 3, the solutions of the problem are developed. Section 4, graphical result and discussion. Finally, conclusions are drawn.

2. PROBLEM FORMULATION

Consider an electrically conducting viscoelastic fluid of second grade under the influence of an externally applied magnetic field and radiative heat transfer between two porous infinite horizontal and parallel plane walls. The distance between the walls, i.e. the channel width, is L. A coordinate system is chosen such that the $x-$ axis is taken along the centre of the channel, and the $y-$ axis is orthogonal to the channel walls, and the origin of the axes is such that the position of the channel walls is $y=0$ and $y=L$, respectively. The wall at $y=0$ is given uniform temperature T_0, while the wall at $y=L$ is subjected to a uniform temperature T_1 where $T_1 > T_0$. The fluid velocity vector $\mathbf{V}=(u,v)$ is assumed to be parallel to the $x-$ axis, so that the component u of the velocity vector does not vanish but the transpiration cross-flow velocity V_0 remains constant, where $V_0<0$ is the injection velocity and $V_0>0$ is the suction velocity.

$$\frac{\partial u}{\partial t}-V_0\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}-\alpha V_0\frac{\partial^3 u}{\partial y^3}+\alpha\frac{\partial^3 u}{\partial y^2\partial t}-\frac{\sigma_e B_0^2 u}{\rho}-\frac{\nu\phi}{K}(1+\frac{\alpha}{\nu}\frac{\partial u}{\partial t})u + g\beta_T(T-T_0) \quad (1)$$

$$\rho C_p\left(\frac{\partial T}{\partial t}-V_0\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}-\frac{\partial q}{\partial y}+Q_0(T-T_0) \quad (2)$$

Subject to the boundary conditions

$$u=0,\ \ T=T_0 \ \text{ on } y=0, \qquad u=0,\ \ T=T_1 \ \text{ on } y=L, \quad (3)$$

where u is the fluid velocity, V_0 is the injection/suction velocity, T is the fluid temperature distribution, p is the pressure, $\alpha=\frac{\alpha_1}{\rho}$ is the second grade parameter, ν is the kinematic viscosity, ρ is the fluid density, σ_e is the fluid electrical conductivity, $B_0(=\mu_e H_0)$ is the electromagnetic induction, μ_e is the magnetic permeability, H_0 is the intensity of the magnetic field, g is the gravitational force, β_T is the coefficient of volume expansion due to temperature, ϕ is the porosity, K is the permeability of the porous medium, C_p is the specific heat at constant pressure, k is the thermal conductivity and Q_0 is the volumetric rate of heat generation/absorption.

The heat generation term in this problem is assumed to be the type given by Foraboschi and Federico (1964).

$$Q=Q_0(T-T_0). \quad (4)$$

where $Q_0>0$ is the heat generation and $Q_0<0$ is the heat absorption.

Following Cogley *et al.* (1968), it is assumed that the fluid is optically thin with a relative low density and the radiative heat flux is given by

$$\frac{\partial q}{\partial y} = 4\alpha^2 (T - T_0). \tag{5}$$

where α is the mean radiation absorption coefficient.

We introduce the following set of non-dimensional quantities:

$$x^* = \frac{x}{L},\quad y^* = \frac{y}{L},\quad u^* = \frac{u}{U_0},\quad t^* = \frac{tU_0}{L},\quad P^* = \frac{LP}{\rho \nu U_0}\quad \theta = \frac{T - T_0}{T_1 - T_0},$$
$$2\gamma = \frac{V_0}{U_0},\quad \beta = \alpha\left(\frac{U_0}{L\nu}\right),\quad H^2 = \frac{L^2 \sigma_e B_0^2}{\rho \nu},\quad Q = \frac{Q_0 L^2}{k},\quad D_a = \frac{K}{\phi L^2}, \tag{6}$$
$$s = \frac{1}{D_a},\quad Gr = \frac{g\beta_T (T_w - T_0) L^2}{\nu U_0},\quad Pe = \frac{U_0 L \rho C_p}{k},\quad N^2 = \frac{4\alpha^2 L^2}{k}.$$

where U_0 is the flow mean velocity.

Using Eq. (6), we can transform Eqs. (1) and (2) into the following non-dimensional form (dropping the * notation)

$$\mathrm{Re}\left(\frac{\partial u}{\partial t} - 2\gamma \frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} - 2\gamma\beta \frac{\partial^3 u}{\partial y^3} + \beta \frac{\partial^3 u}{\partial y^2 \partial t} - \beta s^2 \frac{\partial u}{\partial t}$$
$$-(s^2 + H^2) u + Gr\theta, \tag{7}$$

$$Pe\left(\frac{\partial \theta}{\partial t} - 2\gamma \frac{\partial \theta}{\partial y}\right) = \frac{\partial^2 \theta}{\partial y^2} - N^2 \theta + Q\theta. \tag{8}$$

with the boundary conditions

$$u = 0,\quad \theta = 0 \ \text{ on } y = 0,$$
$$u = 0,\quad \theta = 1 \ \text{ on } y = 1. \tag{9}$$

where Re is the Reynolds number, H is the Hartmann number, γ is the non-dimensional suction/injection velocity, β is the non-dimensional second grade parameter, $s = \frac{1}{D_a}$ is the porous medium shape factor parameter, Gr is the Grashof number, Pe is the Peclet number, N is the radiation parameter and Q is the heat source/sink parameter.

It must be noted that when $\beta = 0$ (Newtonian fluid) and $Q = 0$ (non source of heat generation), Eqs. (7) and (8) are identical to those found by Ahmer *et al*. (2010).

3. SOLUTION METHODOLOGIES

a. Flow assumptions

Consider the solutions of Eqs. (7) and (8) for purely oscillatory flow in the form:

$$u(y,t) = u_0(y) e^{i\omega t}, \tag{10}$$

$$\theta(y,t) = \theta_0(y) e^{i\omega t}, \tag{11}$$

$$-\frac{\partial P}{\partial x} = \lambda e^{i\omega t}. \tag{12}$$

where λ is a real constant, ω is the frequency of the oscillation and $i = \sqrt{-1}$ is the imaginary constant.

Using the flow assumptions Eqs. $(10)-(12)$, Eqs. $(7)-(9)$ becomes

$$(1-i\beta\omega)\frac{d^2u_0(y)}{dy^2}-2\gamma\beta\frac{d^3u_0(y)}{dy^3}+2\gamma\,\mathrm{Re}\frac{du_0(y)}{dy}-m_2^2u_0(y)=-\lambda-Gr\theta_0, \quad (13)$$

$$\frac{d^2\theta_0(y)}{dy^2}+2\gamma Pe\frac{d\theta_0(y)}{dy}-m_1^2\theta_0(y)=0, \quad (14)$$

with boundary conditions

$$\begin{aligned} &u_0=0,\ \ \theta_0=0\ \text{ on } y=0,\\ &u_0=0,\ \ \theta_0=1\ \text{ on } y=1. \end{aligned} \quad (15)$$

where $m_1=\sqrt{N^2+i\omega Pe-Q}$ and $m_2=\sqrt{s^2+H^2+i\omega\left(\mathrm{Re}+\beta s^2\right)}$.

It is appear from Eqs. (13) and (14) that the energy Eq. (14) is uncouple from momentum Eq. (13). Therefore, we could obtain first the solution for fluid temperature $\theta_0(y)$ by fixing Eq. (13) then deploying it in Eq. (13) the solution for fluid velocity $u_0(y)$ can be obtained.

Solving Eq. (14) using the boundary conditions (15), we obtain

$$\theta_0(y)=\frac{1}{e^{d_1}-e^{d_2}}\left(e^{yd_1}-e^{yd_2}\right). \quad (16)$$

Using the assumption given by Eq. (11), the solution Eq. (16) becomes

$$\theta(y)=e^{i\omega t}\left\{\frac{1}{e^{d_1}-e^{d_2}}\left(e^{yd_1}-e^{yd_2}\right)\right\}. \quad (17)$$

where

$$d_{1,2}=-Pe\gamma\mp\sqrt{Pe^2\gamma^2+m_1^2}.$$

Substituting Eq. (16) into Eq. (13), we obtain

$$\begin{aligned} &(1-i\beta\omega)\frac{d^2u_0(y)}{dy^2}-2\gamma\beta\frac{d^3u_0(y)}{dy^3}+2\gamma\,\mathrm{Re}\frac{du_0(y)}{dy}-m_2^2u_0(y)\\ &=-\lambda-\frac{Gr}{e^{d_1}-e^{d_2}}\left(e^{yd_1}-e^{yd_2}\right). \end{aligned} \quad (18)$$

b. Perturbation technique

To solve Eq. (18) subject to the boundary conditions (15), we employ the regular perturbation and expand the function $u_0(y)$ in terms of parameter $\beta(\beta\le 1)$, that is

$$u_0(y)=u_{0,0}(y)+\beta u_{0,1}(y). \quad (19)$$

Substituting Eq. (19) into Eq. (18) and collecting the coefficient of equal powers of β we arrived at the following problems

$$\begin{aligned} &\frac{d^2u_{0,0}}{dy^2}+2\gamma\,\mathrm{Re}\frac{du_{0,0}}{dy}-m_2^2u_{0,0}=-\lambda-\frac{Gr}{e^{d_1}-e^{d_2}}\left(e^{yd_1}-e^{yd_2}\right),\\ &\qquad u_{0,0}(0)=u_{0,0}(1),\ \ 0, \end{aligned} \quad (20)$$

$$\frac{d^2u_{0,1}}{dy^2}+2\gamma\,\mathrm{Re}\frac{du_{0,1}}{dy}-m_2^2u_{0,1}=2\gamma\frac{d^3u_{0,0}}{dy^3}-i\omega\frac{d^2u_{0,0}}{dy^2}, \qquad (21)$$

$$u_{0,1}(0)=u_{0,1}(1),\quad 0.$$

The solutions of the Eqs. (20) and (21) are respectively given by

$$u_{0,0}=A_1e^{yd_3}+A_2e^{yd_4}+\frac{\lambda}{n_2^2}-\frac{Gr}{e^{d_1}-e^{d_2}}\left[\frac{e^{yd_1}}{\{(\mathrm{Re}-Pe)(2\gamma d_1-i\omega)-S^2(1+i\beta\omega)-(H^2+Q-N^2)\}}-\frac{e^{yd_2}}{\{(\mathrm{Re}-Pe)(2\gamma d_2-i\omega)-S^2(1+i\beta\omega)-(H^2+Q-N^2)\}}\right]. \qquad (22)$$

where d_3, d_4, A_1 and A_2 are constant defined by

$$d_{3,4}=-\mathrm{Re}\,\gamma\mp\sqrt{\mathrm{Re}^2\gamma^2+m_2^2}, \qquad (23)$$

$$A_1=-A_2-\frac{\lambda}{m_2^2}+\frac{Gr}{e^{d_1}-e^{d_2}}\left[\frac{4\gamma(\mathrm{Re}-Pe)\sqrt{Pe^2\gamma^2+m_1^2}}{(\mathrm{Re}-Pe)\left\{\begin{array}{c}(\mathrm{Re}-Pe)\left(4\gamma^2(Q-N^2)-\omega^2\right)\\+\left(s^2(1+i\beta\omega)+H^2+Q-N^2\right)\left(4\gamma^2+2i\omega\right)\end{array}\right\}+\left(s^2(1+i\beta\omega)+H^2+Q-N^2\right)^2}\right], \qquad (24)$$

$$A_2=-\frac{\lambda\left(1-e^{d_3}\right)}{m_2^2\left(e^{d_4}-e^{d_3}\right)}+\frac{Gr}{\left(e^{d_1}-e^{d_2}\right)\left(e^{d_4}-e^{d_3}\right)}\left[\frac{(\mathrm{Re}-Pe)\left\{e^{d_1}(2\gamma d_2-i\omega)-e^{d_2}(2\gamma d_1-i\omega)+e^{d_3}\left(4\gamma\sqrt{Pe^2\gamma^2+m_1^2}\right)\right\}-\left\{s^2(1+i\beta\omega)+H^2+Q-N^2\right\}\left(e^{d_1}-e^{d_2}\right)}{(\mathrm{Re}-Pe)\left\{\begin{array}{c}(\mathrm{Re}-Pe)\left(4\gamma^2(Q-N^2)-\omega^2\right)\\+\left(s^2(1+i\beta\omega)+H^2+Q-N^2\right)\left(4\gamma^2+2i\omega\right)\end{array}\right\}+\left(s^2(1+i\beta\omega)+H^2+Q-N^2\right)^2}\right], \qquad (25)$$

and

$$u_{0,1}=B_1e^{yd_3}+B_2e^{yd_4}+yB_3e^{yd_3}+yB_4e^{yd_4}+B_5e^{yd_1}+B_6e^{yd_2}. \qquad (26)$$

where B_1, B_2, B_3, B_5 and B_6 are constant defined by

$$B_1=-(B_2+B_5+B_6), \qquad (27)$$

$$B_2=-\frac{1}{\left(e^{d_4}-e^{d_3}\right)}\left\{B_3e^{d_3}+B_4e^{d_4}+B_5\left(e^{d_1}-e^{d_3}\right)+B_6\left(e^{d_2}-e^{d_3}\right)\right\}, \qquad (28)$$

$$B_3=\frac{A_1}{2}\left\{d_3^2(i\omega-2\gamma d_3)\left(\gamma^2\,\mathrm{Re}^2+m_2^2\right)^{-\frac{1}{2}}\right\}, \qquad (29)$$

$$B_4=-\frac{A_2}{2}\left\{d_4^2(i\omega-2\gamma d_4)\left(\gamma^2\,\mathrm{Re}^2+m_2^2\right)^{-\frac{1}{2}}\right\}, \qquad (30)$$

$$B_5=-\frac{Gr}{\left(e^{d_1}-e^{d_2}\right)}$$
$$\left\{\frac{d_1^2\left(i\omega-2\gamma d_1\right)}{\left[\left(\mathrm{Re}-Pe\right)2\gamma d_1-m_2^2\right]\left[\left(\mathrm{Re}-Pe\right)\left(2\gamma d_1-i\omega\right)-s^2\left(1+i\beta\omega\right)-\left(H^2+Q-N^2\right)\right]}\right\}, \tag{31}$$

$$B_5=\frac{Gr}{\left(e^{d_1}-e^{d_2}\right)}$$
$$\left\{\frac{d_2^2\left(i\omega-2\gamma d_2\right)}{\left[\left(\mathrm{Re}-Pe\right)2\gamma d_2-m_2^2\right]\left[\left(\mathrm{Re}-Pe\right)\left(2\gamma d_2-i\omega\right)-s^2\left(1+i\beta\omega\right)-\left(H^2+Q-N^2\right)\right]}\right\}. \tag{32}$$

Therefore, the required solution is

$$u_0(y)=A_1e^{yd_3}+A_2e^{yd_4}+\frac{\lambda}{n_2^2}-\frac{Gr}{e^{d_1}-e^{d_2}}\left[\begin{array}{l}\frac{e^{yd_1}}{\left\{\left(\mathrm{Re}-Pe\right)\left(2\gamma d_1-i\omega\right)-s^2\left(1+i\beta\omega\right)-\left(H^2+Q-N^2\right)\right\}}\\ -\frac{e^{yd_2}}{\left\{\left(\mathrm{Re}-Pe\right)\left(2\gamma d_2-i\omega\right)-s^2\left(1+i\beta\omega\right)-\left(H^2+Q-N^2\right)\right\}}\end{array}\right] \tag{33}$$
$$+\beta\left[B_1e^{yd_3}+B_2e^{yd_4}+yB_3e^{yd_3}+yB_4e^{yd_4}+B_5e^{yd_1}+B_6e^{yd_2}\right].$$

Using the assumption given by Eq. (10), the solution Eq. (33) becomes

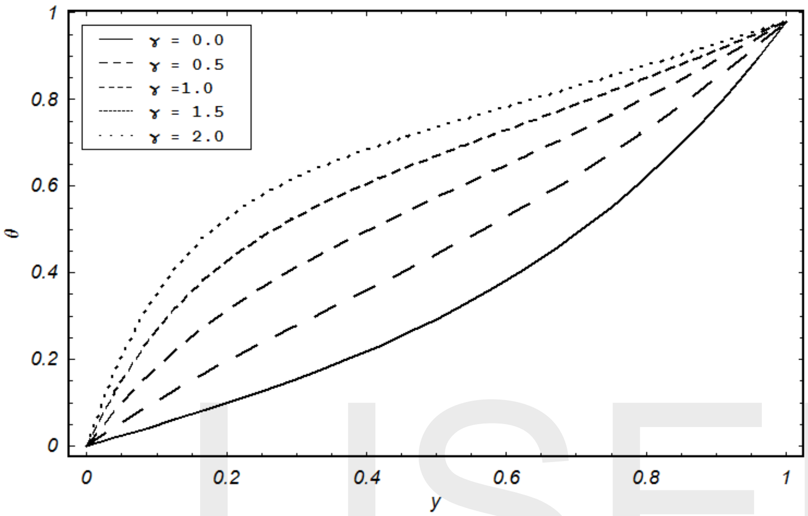
$$u.(y)=e^{i\omega t}\left\{A_1e^{yd_3}+A_2e^{yd_4}+\frac{\lambda}{n_2^2}-\frac{Gr}{e^{d_1}-e^{d_2}}\left[\begin{array}{l}\frac{e^{yd_1}}{\left\{\left(\mathrm{Re}-Pe\right)\left(2\gamma d_1-i\omega\right)-s^2\left(1+i\beta\omega\right)-\left(H^2+Q-N^2\right)\right\}}\\ -\frac{e^{yd_2}}{\left\{\left(\mathrm{Re}-Pe\right)\left(2\gamma d_2-i\omega\right)-s^2\left(1+i\beta\omega\right)-\left(H^2+Q-N^2\right)\right\}}\end{array}\right]\right. \tag{34}$$
$$\left.+\beta\left[B_1e^{yd_3}+B_2e^{yd_4}+yB_3e^{yd_3}+yB_4e^{yd_4}+B_5e^{yd_1}+B_6e^{yd_2}\right]\right\}.$$

4. RESULT AND DISCUSSIONS

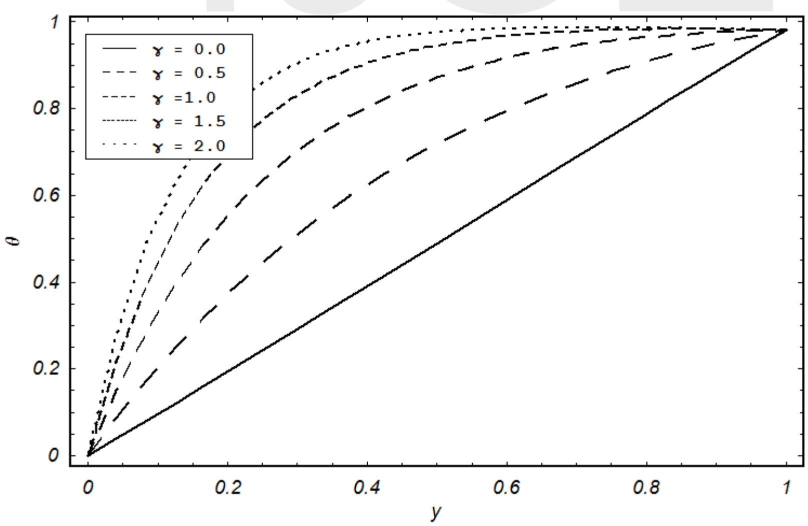
In this section, the variations of fluid velocity and temperature distribution due to oscillatory nature of the flow are presented graphically and discussed over various immerging flow parameters. The temperature distribution given by Eq. (17) are displayed in Figs. 1-4 while the velocity field given by Eq. (34) are displayed in Fig. 5-7 versus the boundary layer coordinate y. In all these figures, we consider only the real part of the velocity and temperature fields.

Figure 1 shows the temperature distribution across the channel with no heat source/sink. These results are identical to those found by Ahmer *et al*. (2010). The velocity profiles when the heat source/sink was considered are shown in Figure. 2 and 3. As can be seen, the heat source has the clear effect of increasing the temperature distribution across the channel, but the thermal radiation weakens the strengh of the temperature distribution. When heat sink presented in Figure 3, we should expect the reverse effect since the energy transmitted to the fluid by the wall is sucked away. Figure 4 illustrates the influence of Peclet number (Pe) on fluid temperature for both injection and suction. It is noticed from Fig. 4 that fluid temperature θ decreases on increasing Pe in the boundary layer region for injection while increases for suction case. Since Pe signifies the relative importance of advection to diffusion. This implies that thermal diffusion tends to increase fluid temperature for suction case while decrease for injection case.

The effects of injection/suction on the second grade fluid motion are shown in Figure 5. This diagram is plotted for different values of the second grade parameter β. It is noted that the influence of the parameter β on the fluid motion depends on injection/suction velocity. Also we can compare the velocity of second grade fluid with velocity corresponding to Newtonian fluid $(\beta=0)$. For injection case, the Newtonian fluid with transpiration flows faster than second grade fluid with transpiration. For suction case the second grade fluid is faster near the Newtonian fluid across the channel. Figure 6 shows the effect of the Reynold's number Re and Grashoff number number Gr on the fluid velocity. It is observed that velocity decreases on increasing the Reynolds's number while increases by increasing the Grashoff number. The effect of Grashof number Gr is quite opposite to that of Re. Fig. 7 shows the effect of the Hartmann number H and porous medium shape factor s on the velocity field. The effect of Hartmann number is quite similar to that of porous medium shape factor.

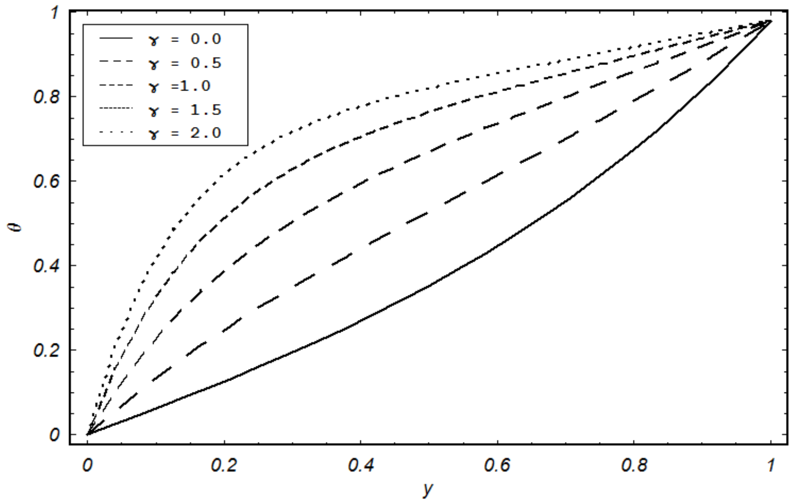


$(N = 2)$

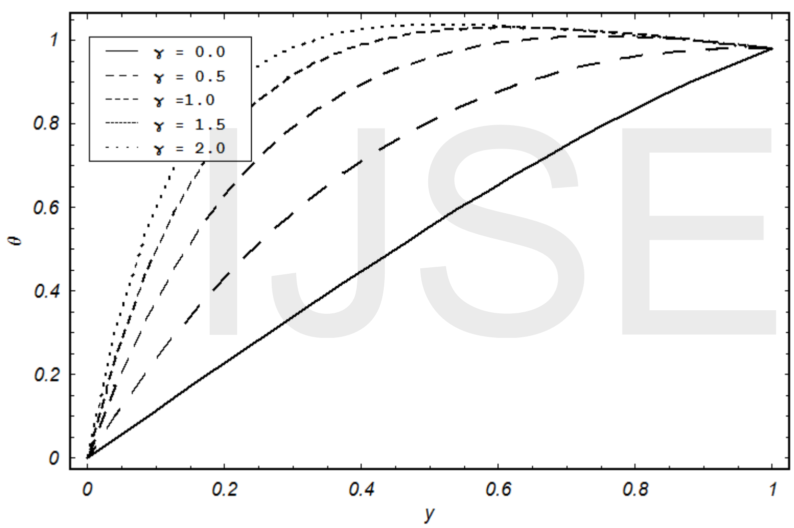


$(N = 0)$

Figure 1 Temperature profile in the case of suction when $Q = 0$

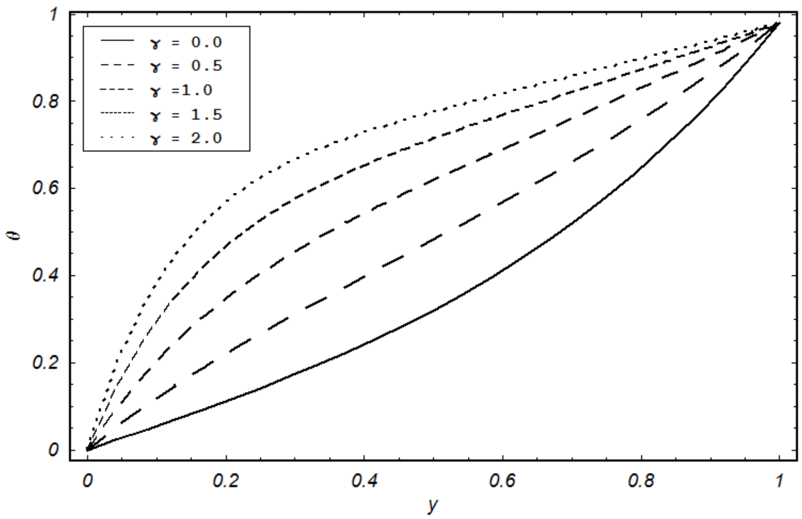


$(N = 2)$

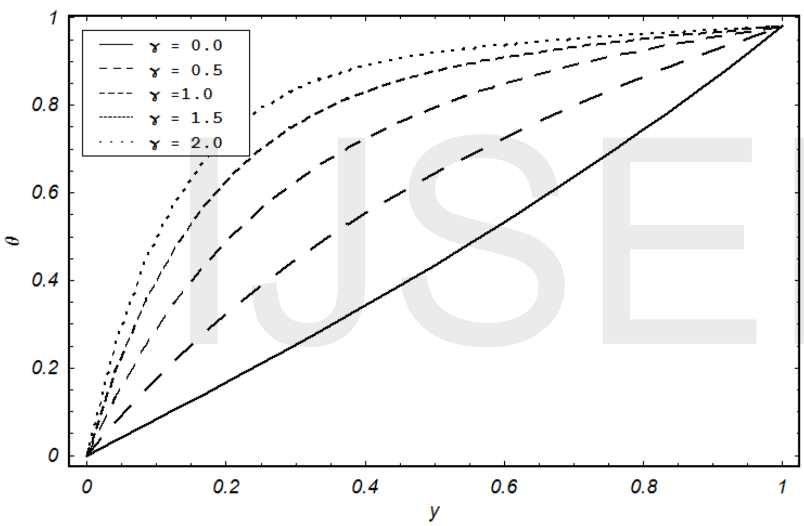


$(N = 0)$

Figure 2 Temperature profile in the case of suction when $Q = 1$ (heat generation)

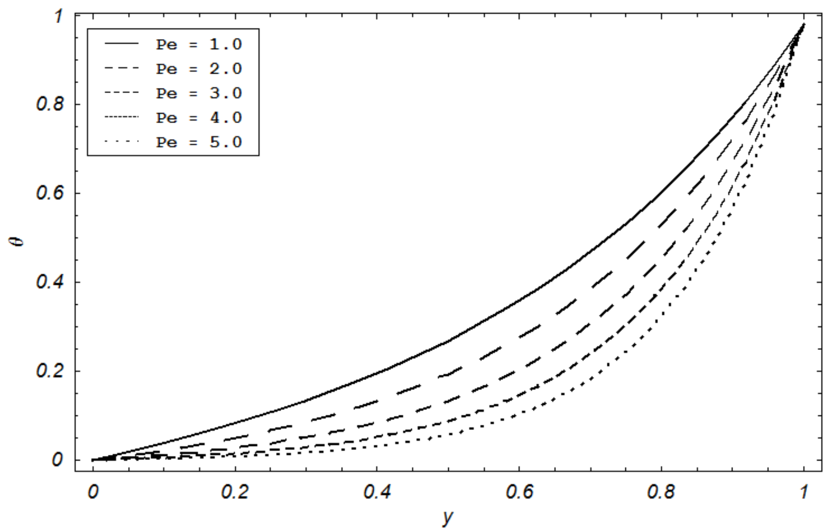


$(N=2)$

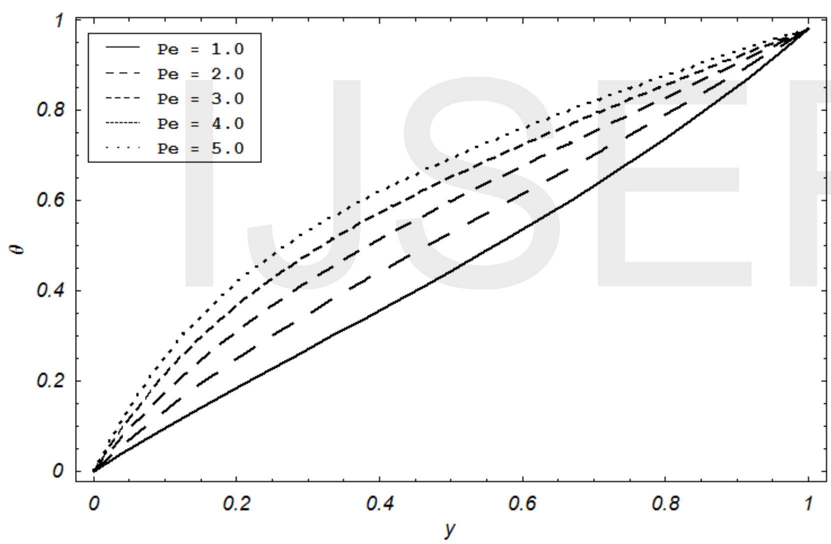


$(N=0)$

Figure 3 Temperature profile in the case of suction when $Q=-1$ (heat sink)

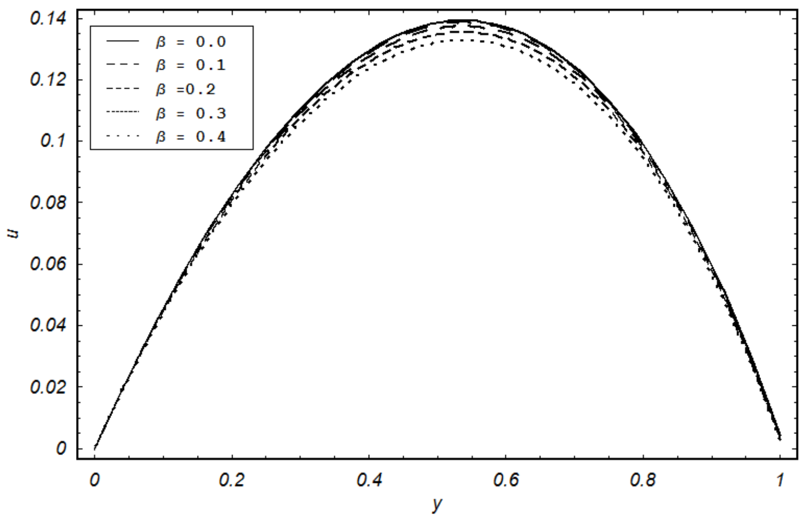


(Injection)

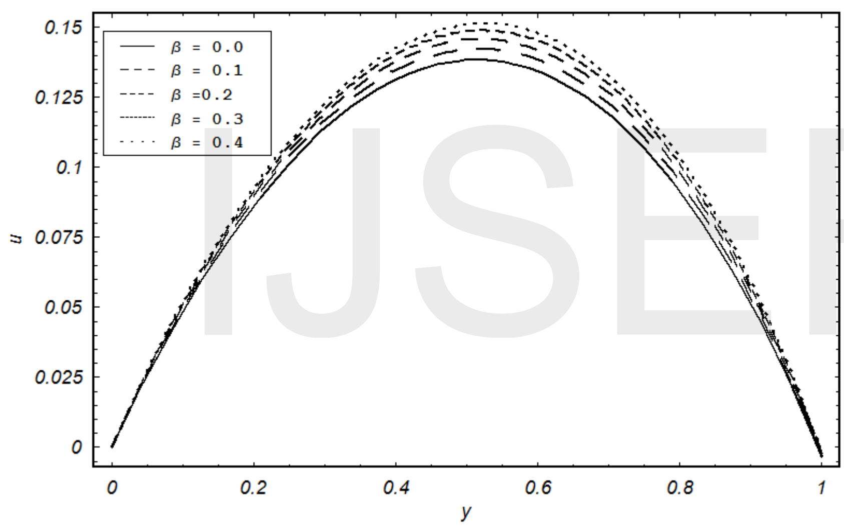


(Suction)

Figure 4 Temperature profile for various values of Peclet number (Pe)



(Injection)



(Suction)

Figure 5 Velocity profile for various second grade parameter β

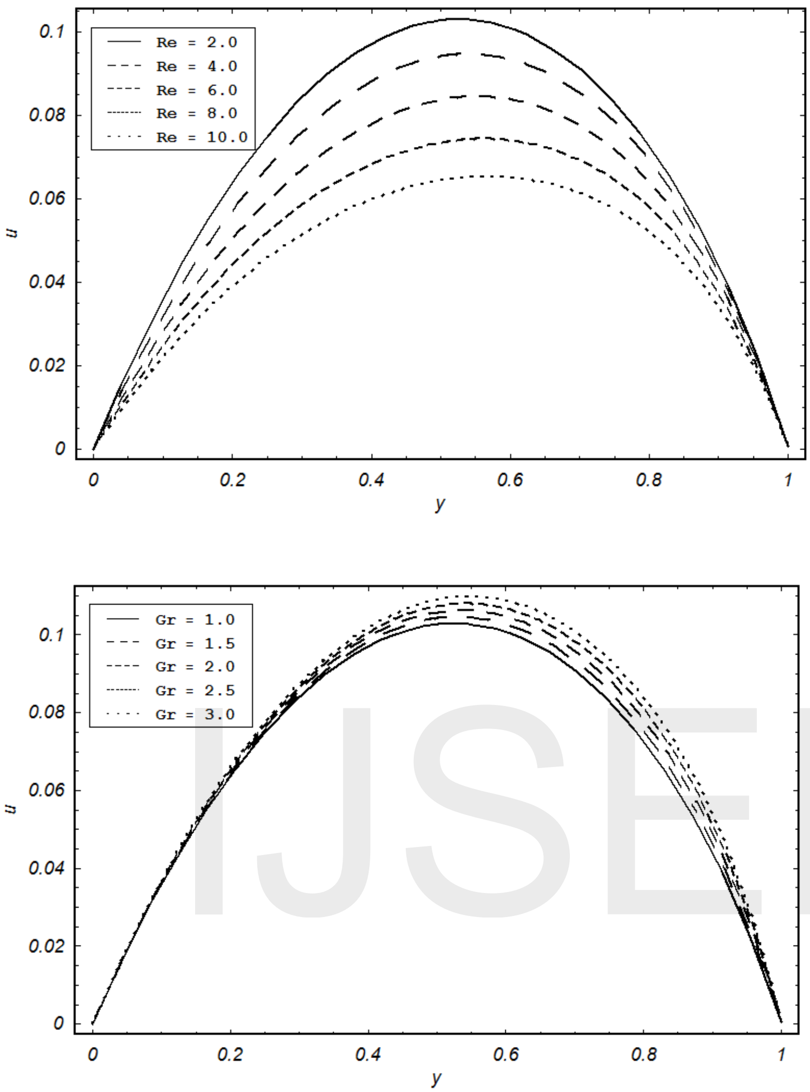
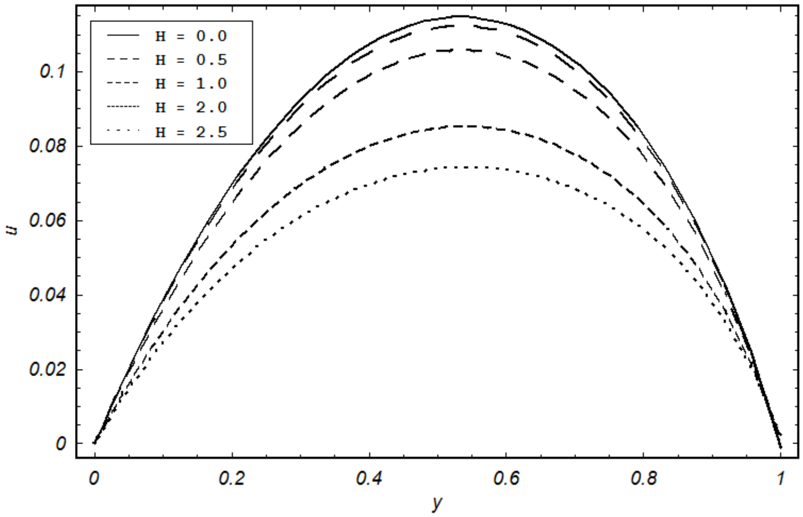


Figure 6 Velocity profile for various of Reynold's number (Re) and Grashoff number (Gr)



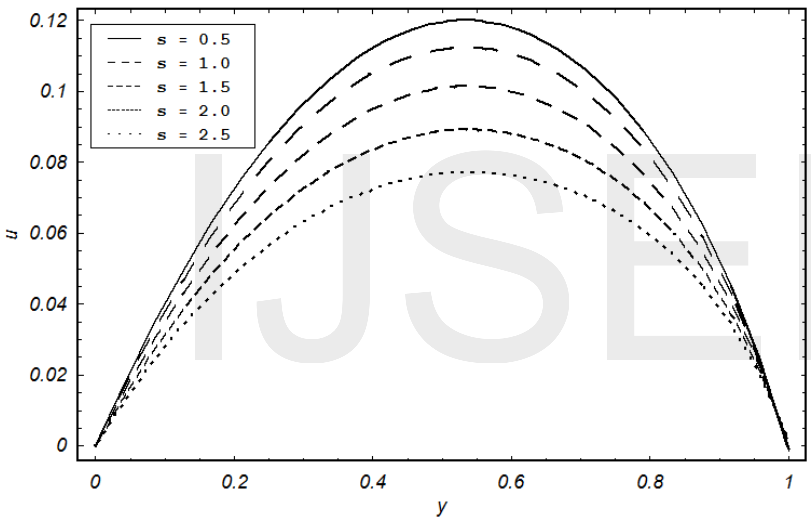


Figure 7 Velocity profile for various values of Hartmann number (H) and porous medium shape factor parameter (s)

5. CONCLUSION

Analytical solutions for the fluid velocity and the temperature distribution have been found for the heat transfer flows of a second grade fluid in a porous channel with heat generation. It is observed that the injection causes an increase in the velocity across the channel, but it reduces the heat transfer phenomenon across the channel. The effects of injection velocity are totally reversed to the suction effects. The thermal radiation, Reynold's number, heat generation and Peclet number are found to reduce the injection effects.

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